

# **Broadband, Range-Dependent Normal Mode Modeling**

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## **LONG-TERM GOAL**

Our long-term goals are to develop practical approaches to modeling broadband acoustic propagation in range-dependent environments using normal mode theory, to improve the capabilities and efficiency of a specific acousto-elastic normal mode model (ORCA), and to provide support to others in the underwater acoustic community who use our modeling products.

## **OBJECTIVES**

Work during FY98 focused on three areas: (1) developing, analyzing, and implementing a method for eliminating the branch line integral from the normal mode solution, (2) investigating the accuracy and characteristics of various adiabatic- and coupled-mode approaches for range-dependent benchmark problems, and (3) completing the documentation of work related to the ONR-sponsored Geoacoustic Inversion Workshop from the previous fiscal year.

## **APPROACH**

The technical approach for eliminating the branch line integral from the normal mode solution was to replace the homogeneous halfspace with one that included an attenuation gradient and to find the modes that effectively replaced the Pekeris branch line integral. Evan Westwood and Robert Koch collaborated in this area. The adiabatic- and coupled-mode approaches have been analyzed by David Knobles and Robert Koch by applying them to the 1986 ASA benchmark problems. Documentation of the geoacoustic inversion results was performed primarily by David Knobles.

## **WORK COMPLETED**

After arriving at the idea to insert a gradient into the lower halfspace in order to eliminate the branch line integral, we derived the reflection coefficient for such a halfspace and analytically studied the location of the modes in the complex  $k$  plane and the nature of their depth-dependent mode functions for the simplest case of a gradient halfspace below a pressure-release interface. A major part of the work was to implement the concept into the existing ORCA normal mode model, which involved computing a different (non-zero) reflection coefficient for the lower halfspace, detecting whether a found mode belonged to the series of "branch line modes" (BLMs), finding the BLMs in the most efficient manner, and incorporating BLMs into the existing broadband algorithm. We presented three papers at the Seattle ICAIASA meeting related to the subject-4 and wrote a journal article for submission to JASA. 5

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The accuracy of the various adiabatic- and coupled-mode approaches was investigated by implementing the theoretical formulations into computer programs and comparing the results to the benchmark solutions. Results were presented at the ICA/ASA,' and are summarized below.

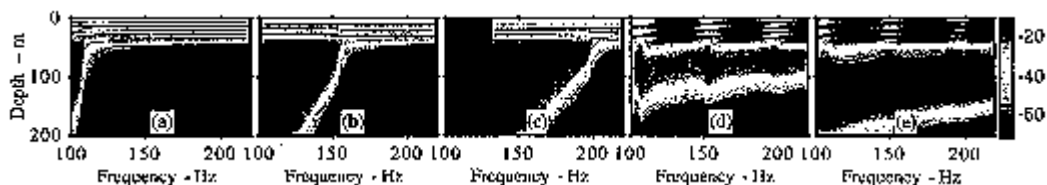
Documentation of FY97 geoacoustic inversion work was completed and is to be published in Ref. 6.

## RESULTS

The significance of the work on branch line modes is summarized below. The branch line integral is eliminated by replacing the homogeneous halfspace with one having a (complex) sound speed gradient. The branch line integral is replaced by a set of modes whose trajectory and spacing in the complex  $k$  plane depend on the magnitude and direction of the complex gradient. For a small, positive *sound speed* gradient, the branch line modes correspond to the former EJP branch cut. For a small, positive *attenuation* gradient, the branch line modes lie on a curve that leaves the former branch point vertically and that corresponds approximately to the former Pekeris branch cut. The total number of modes is minimized by the latter choice of gradient, in which case the modal spectrum consists of a finite number of trapped modes and infinite numbers of leaky and branch line modes. (The number of modes that are needed depends on the minimum range at which an accurate field is required.) Except when a mode lies near the former branch line, the trapped and leaky modes for a small-gradient halfspace correspond closely with those for a homogeneous halfspace. Larger gradients result in fewer branch line modes but larger errors (compared to the homogeneous halfspace) in the pressure field as range increases.

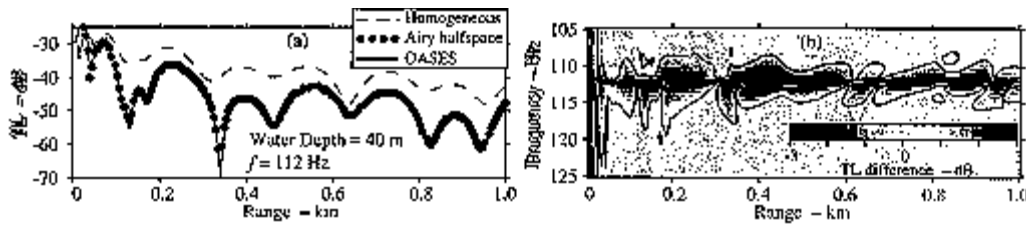
Modes for the gradient halfspace problem are found using the same method as that used in the ORCA nonnal mode model: the upward- and downward-looking reflection coefficients are computed at a reference depth, and the  $R_1 R_2 = 1$  contour is followed in the complex  $k$  plane.<sup>1</sup> The usual trapped and leaky modes are found by placing the reference depth in the water column. The branch line modes, which appear as island modes for the usual choice of reference depth, are found on the main  $R_1 R_2 = 1$  contour when the reference depth is set to the top of the lower halfspace.

Characteristics of the branch line modes, such as the frequency dependence of their eigenvalues, depth functions (see Fig. 1), and group velocities, have been examined in detail in Ref. 5. The behavior is consistent with the notion that the branch line modes represent the resonant behavior of the lateral wave component of the field: the eigenvalues of the branch line modes correspond to energy near the critical angle; their mode functions are large in the halfspace and peak up in the water column only when a trapped mode passes below them in the complex  $k$  plane [see Fig. 1 (d)-(e)]; and their group velocities lie near the halfspace sound speed.

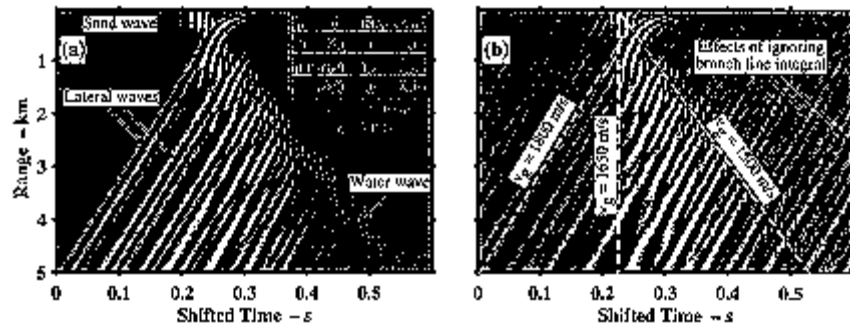


**Figure 1.** Mode function magnitudes in dB imaged versus depth and frequency for a Pekeris waveguide with an gradient halfspace: (a)-(c) modes 3-5, (d)-(e) branch line modes 1 and 2.

In Figs. 2 and 3, broadband calculations using gradient halfspaces are presented in both the frequency and time domains. Figure 2(a) shows that, when a mode is near cutoff [in this case, mode 3 at 112 Hz - see Fig. 1 (a)] the homogeneous halfspace TL is incorrect at all ranges and that the gradient halfspace corrects the problem. Figure 2(b) shows the error in TL between the gradient and homogeneous halfspace treatments versus range and frequency. The frequency interval over which the branch line modes are significant is about 3-5 Hz, and these intervals occur every 45 Hz (see Fig. 1). Figure 3 shows images of time series versus range for an environment (see inset) in which lateral waves are the primary mechanism of propagation. The inclusion of the branch line integral by way of the gradient halfspace in (a) produces the correct lateral wave behavior, whereas ignoring the branch line integral causes non-causal arrivals with lateral wave group speed in (b).



**Figure 2. Transmission loss comparisons for a 40-m Pekeris waveguide. (a) TL versus range at 112-Hz; (b) TL differences between the gradient-halfspace and homogeneous-halfspace mode solutions.**



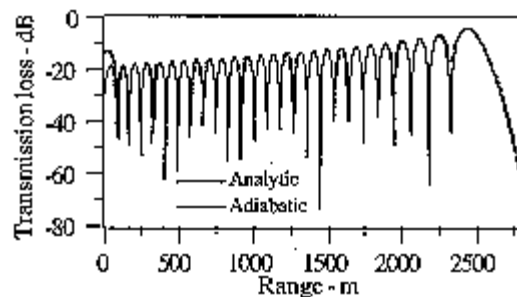
**Figure 3. Normalized pressure time series versus range for a 50-Hz pulse in 10 m of water with a 100-m lossy sand bottom layer and a lossless lower halfspace: (a) using a gradient halfspace, and (b) using a homogeneous halfspace. Use of the gradient halfspace correctly accounts for the lateral wave energy.**

## THE FOLLOWING LESSONS WERE LEARNED FROM THE WORK ON ADIABATIC AND COUPLED-MODE CALCULATIONS:•

- Proper mode association (the pairing of modes from neighboring mode sets as one marches out in range) is critical for adiabatic, range-dependent calculations and is not straightforward when multiple ducts or branch line modes are present. For example, when a trapped mode moves past the branch point and becomes a leaky mode, its mode number, if given according to decreasing  $\text{Re}(k)$ , changes because it passes by the stationary branch line modes. Proper mode association in this case

could be achieved by keeping the BLMs in a separate list. Modes can also change order when two or more ducts exist and a family of modes is trapped in each one. Improper association would result in energy trapped in one duct to suddenly jump into the another duct.

- Computing leaky modes constitutes an efficient method, compared to false-bottom techniques, for including energy that is steeper than the halfspace critical angle in normal mode calculations.
- Adiabatic TL calculations using trapped and leaky modes (TM+LM) are accurate for the lossy, penetrable-wedge ASA benchmark problem (using either a homogeneous or gradient halfspace). Although the *amplitude* of the field is correct, the *phase* of the field has a linearly-increasing error in depth, which is the difference between the adiabatic *vertical* modes and the adiabatic *wedge* modes (see Ref. 8 for more details on wedge modes).
- For the *lossless*, penetrable-wedge benchmark problem (using a homogeneous halfspace), the adiabatic TM+LM treatment exhibits errors because the modal transition from trapped-to-leaky is not smooth. When a gradient halfspace is used and the branch line modes are included, the results are much more accurate. (Here, proper mode association is required.) However, it appears that most of the accuracy gain is achieved by smoothing the trapped-leaky transition because even without the BLMs the accuracy of the gradient halfspace solution is good.
- A one-way, marching type of coupled-mode model using the TM+LM+BLM decomposition works well on the lossy wedge. Not only is the TL correct, but the phase is correct as well.
- A *two-way* adiabatic normal mode method based on the endpoint method is accurate for the pressure-release-bottom ASA benchmark wedge problem, IO which is demonstrated in Fig. 4, and correctly describes, *without coupling*, the significant backscatter.



**Figure 4. Comparison of two-way adiabatic mode results with analytic reference solution for 10-Hz version of ASA benchmark 1. 1 0**

- The adiabatic solution required in the differential form of the coupled-mode solution I may be replaced by the WKB form without loss of accuracy.<sup>4</sup>
- Perturbative expansion solutions (Lanczos series, Born series, and effective coupling operator) were obtained for sample ASA benchmarks and were found to converge to the exact coupled results.

## IMPACT/APPLICATIONS

Most practical range-dependent acoustic propagation models are based on either the PE method or on normal mode theory. Reasons for continuing to pursue research in the area of normal mode theory include:

- Coupled-mode models, although currently too slow for most practical applications, can provide benchmark solutions to evaluate PE solutions, the accuracy of which can be sensitive to sampling and false bottom parameters.
- Adiabatic mode approaches are more efficient than PE when only a sparse set of source and receiver depths are required.
- In situations where mode coupling is important, the perturbative techniques mentioned above or wedge mode techniques may be viable methods for including first-order coupling effects in an efficient adiabatic approach.
- Normal mode theory provides quantities such as arrival times, multipath time spread, and vertical arrival angles that are difficult to estimate using PE methods.

Regarding the research on eliminating the branch line integral, the gradient halfspace approach should also be useful for *range-dependent* non-nal mode calculations based on adiabatic or coupled-mode theory. Mode cutoff due to decreasing frequency is analogous to mode cutoff due to decreasing water depth, for example. Adiabatic treatment of the problem would be best achieved by keeping track of the branch line modes separately so that a trapped mode passing through cutoff transfers its energy to a leaky mode, rather than to a branch line mode. In reality, we would expect some degree of coupling at such a transition because the mode function of the trapped/leaky mode has a shape similar to those of the branch line modes. Another attractive aspect of the gradient halfspace approach for coupledmode calculations is that the leaky modes eventually decay with depth, making the overlap integrals well defined. If the integrals can be solved analytically using Airy function identities, then the coupling calculations would be very efficient.

An important observation from our application of adiabatic- and coupled-mode theory to the ASA benchmark wedge problems is that an appropriate adiabatic treatment of those problems gives nearly exact results. Therefore we would recommend the development of a new set of benchmarks that would involve more complex range variability such that mode coupling was inherently important.

## TRANSITIONS

Software and understanding gained under this work has continued to provide input into the enhancement of a broadband adiabatic mode model (NAUTILUS7) that is part of SPARS, a performance prediction tool developed for the Navy by ARL:UT.

We continue to make the ORCA normal mode model available to the community and to provide support to users when needed. We have been informed of use of the model at the following institutions: NRAD (Abawi's coupled mode calculations), MPL (part of genetic algorithm code SAGA); NAWC, U. of Hawaii (whale research); SACLANT (part of broadband, range-dependent code PROSIM; reverberations calculations); NRL SSC; U. of Victoria (ocean bottom parameter sensitivity studies); MIT/WHOI; University of Bochum (Germany); and Pennsylvania State University.

During the past year ORCA was used as a "mode engine" for a modeling tool that was developed by ARL:UT to provide range-dependent TL in real-world environments. The tool is part of a larger software package put together for the N87 program office for evaluating active sonar systems. Bathymetry and sound speed profiles for an area are obtained from the Web, the area is provinced, mode characteristics are computed, and adiabatic mode calculations are performed along radials.

## RELATED PROJECTS

None.

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- D. P. Knobles, R. A. Koch, E. K. Westwood, and T. Udagawa, "The inversion of ocean waveguide parameters using a nonlinear least squares approach," *J. Comp. Acoust.* **6** (1998) (in press).
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